B. Sc. Part I Physics

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Galilean Transformation of the velocity of a Particle :–

Consider two systems S & S'. S' is moving with velocity $\vec{V} = \hat{i} V_x + \hat{j} V_y + \hat{k} V_z$ relative to S. If $\vec{r}(t)$ and $\vec{r}'(t')$ are the co-ordinates of any particle as observed by observer in system S & S'. Then the Galilean transformation equations of space and time may be expressed as.

$$\left.\begin{aligned} \vec{r}' &= \vec{r} - \vec{V}t \quad \text{— (A)} \\ t' &= t \quad \text{— (B)} \end{aligned}\right\} \text{—— (1)}$$

Now differentiating equ^n (1A) with respect to t, we get

$$\frac{d\vec{r}'}{dt} = \frac{d}{dt}(\vec{r} - \vec{V}t)$$

$$= \frac{d\vec{r}}{dt} - \frac{d}{dt}(\vec{V}t)$$

$$\frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \vec{V} \quad \text{——— (2)}$$

from equ^n (1B) $dt' = dt$

∴ from equ^n (2) $\dfrac{d\vec{r}'}{dt'} = \dfrac{d\vec{r}}{dt} - \vec{V}$

or, $$\boxed{\vec{u}' = \vec{u} - \vec{V}} \quad \text{—— (3)}$$

Where $\dfrac{d\vec{r}'}{dt'} = \vec{u}'$ = velocity vector of particle in S'.

and $\dfrac{d\vec{r}}{dt} = \vec{u}$ = velocity vector of particle in S.

equ^n (3) represents the Galilean transformation of the velocity of the particle =0

Teacher's Signature:

Galilean transformation of the accelerations of the Particle :-

we have Galilean transformation for velocity of Particle is

$$\vec{u}' = \vec{u} - \vec{v}$$

Differentiating above equ^n with respect to t, we get,

$$\frac{d\vec{u}'}{dt} = \frac{d}{dt}(\vec{u} - \vec{v})$$

$$= \frac{d\vec{u}}{dt} - \frac{d\vec{v}}{dt} = 0$$

or; $\frac{d\vec{u}'}{dt} = \frac{d\vec{u}}{dt}$ or $\frac{d\vec{u}'}{dt'} = \frac{d\vec{u}}{dt}$

Since $dt' = dt$

or $\vec{a}' = \vec{a}$

where $\vec{a}' = \frac{d\vec{u}'}{dt}$ = Rate of change of velocity of Particle in System S'

= acceleration of the Particle in S'.

and $\vec{a} = \frac{d\vec{u}}{dt}$ = Rate of change of velocity of Particle in System S.

= acceleration of the Particle in System S

Thus the acceleration observed by the observer in different inertial frame is same, i.e, acceleration is invariant under Galilean transformation.

As mass is also invariant

So, $\vec{F} = m\vec{a}$

is invariant in inertial frame

In other words we may say that Newton's Second law is valid in every inertial system, i.e. it is invariant under Galilean transformation.

Example— Show That Newtonian fundamental equn are invariant under Galilean transformation.

Soln :-

From newton's 2nd law of motion. In the absolute system of coordinate the particle acted upon by a force having an acceleration $\frac{d^2x}{dt^2}$ is given by

$$F = m \frac{d^2x}{dt^2} \quad \text{—— (1)}$$

Now according to Galilean transformation

$$u'_x = u - v$$

$$\frac{dx'}{dt'} = \frac{du}{dt} - v$$

Differentiating equ^n we get

$$\frac{d^2x'}{dt'^2} = \frac{d^2x}{dt^2} = 0$$

In Newtonian mechanics since force and masses are absolute quantities.

So, $F = F'$

$m = m'$

$$\therefore \quad F' = m' \frac{d^2x'}{dt^2}$$

i.e. w.r.t. other frame of reference the second law has the same form. i.e. 2nd law is valid for every inertial system.

Thus we may say that Newtonian fundamental equations are invariant under Galilean transformations.